Gas-Liquid Contactors," *Ind. Eng. Chem. Process Design Develop.*, 5, 15 (1966).
Karwat, H., "Verteilung von Gasen in Flüssigkeiten durch Rührer," *Chem. Ing. Technik*, 31, 588 (1959).
Laurent, A., and J. C. Charpentier, "Détermination par Méthode Chimique des Aires Interfaciales et des Coefficients de Transfert de Matière dans les Différents Types d'Absorbeurs et de Réacteurs Gaz-Liquide," *J. Chimie Physique*, 2, 336 (1975).
Levich, V. G., *Physical Hydrodynamics*, Prentice Hall, Englewood Cliffs, N.J. (1962).
Levich, V. G., "Movement of Drops and Bubbles in Liquid Media," *Intern. Chem. Eng.*, 2, 78 (1962).
Linek, V., and J. Mayrhoferova, "The Chemical Method for the Determination of Interfacial Area," *Chem. Eng. Sci.*, 24, 481 (1969).
Liu, M. S., M. R. Branion, and D. W. Duncan, "Oxygen Transfer to Water and to Sodium Sulphite Solutions," *J. W.P.C.F.*, 44, 34 (1972).
Mehta, V. D., and M. M. Sharma, "Mass Transfer in Mechanically Agitated Gas-Liquid Reactor," *Chem. Eng. Sci.*, 26, 461 (1971).
Midoux, N., "Les Réacteurs Gas-Liquide à Cuve Agitée," Thèse, I.N.P.L., Nancy, France (1978).
Padmanabhan, L., and B. Gal-Or, "Analysis of the Effect of Particle Size and Residence Time Distributions on Heat and Mass Transfer with Linear Source in Particulate Systems," *Chem. Eng. Sci.*, 23, 631 (1968).
Reith, T., "Physical Aspects of Bubble Dispersions in Liquids," Ph.D. thesis, Univ. Delft, Holland (1968).
Sridharan, K., "Studies in Mass Transfer," Ph.D. thesis, Univ. Bombay, India (1975).
Van't Riet, K., and J. M. Smith, "The Behaviour of Gas-Liquid Mixtures Near Rushton Turbine Blades," *Chem. Eng. Sci.*, 28, 1031 (1973).
Westerterp, K. R., "Design of Agitators for Gas-Liquid Contacting," *ibid.*, 18, 495 (1963).
Westerterp, K. R., L. L. Van Dierendonck, and J. A. de Kraa, "Interfacial Areas in Agitated Gas-Liquid Contactors," *ibid.*, 157 (1963).
Yagi, H., and F. Yoshida, "Desorption of CO_2 from Fermentation Broth," *Biotechnol. Bioeng.*, 19, 801 (1977).
Yoshida, F., A. Ikeda, S. Imakawa, and Y. Miura, "Oxygen Absorption Rates in Stirred Gas-Liquid Contactors," *Ind. Eng. Chem.*, 52, 435 (1960).

Manuscript received August 14, 1978; revision received December 5, and accepted January 11, 1979.

Velocity Profiles of Viscoelastic Fluids at the Inlet of an Annulus

PHILIP K. L. TAN and CARLOS TIU

Department of Chemical Engineering
Monash University
Clayton, Victoria, 3168
Australia

Most entry flow problems have been analyzed using boundary-layer approximations. Inherent in the analysis is the assumption of a uniform (flat) velocity profile at the inlet plane. In practice, this condition may not be achieved for viscoelastic fluids unless it is artificially generated. For example, Brocklebank and Smith (1970) and Tan and Tiu (1977) both used a flow distributor upstream of the entry to generate a flat profile in a pipe and an annulus, respectively. For many polymer melts and solutions, flow patterns upstream and downstream of the contraction are strongly influenced by the elastic properties of the fluids (Tordella, 1957; Ballenger and White, 1970; Cable and Boger, 1978). Thus, the inlet velocity profile could attain any shape other than flat for viscoelastic fluids, depending upon the fluid characteristics. Furthermore, conflicting experimental results have been reported concerning the effects of elasticity on the entry length and on the excess pressure drop (Sutterby, 1965; Boger and Ramamurthy, 1972; Busby and MacSporran, 1976). The inconsistency could be attributed to the different inlet velocity conditions employed by these workers in determining the entrance length. This paper aims to resolve experimentally the inlet velocity conditions for viscoelastic fluids flowing through an annulus. Two inlet geometries are studied: an abrupt contraction from a large to a small annulus (2:1 contraction ratio) and a smooth entry through a conical section upstream of the contraction. In both cases, no flow distributor is used in the upstream side. The flow approaches the contraction in its fully developed state.

EXPERIMENT RESULTS AND DISCUSSION

The experimental setup and procedure for velocity profile measurements in the annulus (aspect ratio of 0.42) have been presented in detail elsewhere (Tiu and Bhattacharyya, 1974; Tan and Tiu, 1977). The only alteration in the experimental equipment was the removal of the flow straightener, and also the conical entry section for the abrupt entry work, from the upstream annulus. A technique employing streak photography was used for point velocity measurements. The test fluids were dilute aqueous solutions of Separan AP30 and MG500 (partially hydrolyzed polyacrylamide, Dow Chemical).

The viscometric functions τ and N_1 were measured for all test fluids on an R16 Weissenberg rheogoniometer over the shear rate range $4 \leq \dot{\gamma} \leq 1\,112$ s^{-1}. Power law models were used to fit the viscometric data:

$$\tau = K\dot{\gamma}^{n} \quad (1)$$

and

$$N_1 = P_{11} - P_{22} = \sigma\dot{\gamma}^{s} \quad (2)$$

In instances where the flow curve was not linear on a log-log plot over the entire shear rate range, it was approximated with two power law regions. Fluid parameters evaluated at the same shear rates as those encountered in each experiment were used in the data analysis. Under the experimental conditions, values of n varied from 0.34 to 0.51, and s from 0.611 to 0.875. Two relevant dimensionless groups, Reynolds number Re and elasticity number ξ, used in the analysis of data were in the range $2.53 \leq Re \leq 973$ and $0.0076 \leq \xi \leq 1.761$, respectively. The parameter ξ is considered to give a better measure of fluid elasticity for the present purpose than Weissenberg number, Deborah number, relaxation time, or stress ratio. In the present context, a viscoelastic fluid is considered to be weakly or strongly elastic depending on the relative magnitude of elastic to inertial forces in the actual flow system.

The accuracy of point velocity measurements was established in two ways. Firstly, the fully developed velocity profile measured beyond the entrance region was compared with the theoretical solution for the flow of power law fluids in an annulus (Fredrickson and Bird, 1958). The validity of Fredrickson and Bird's theoretical solution has already been verified previously (Tiu and Bhattacharyya, 1974). The failure of power law solution in predicting pressure drop at low flow rate in annuli, as shown by Vaughn and Bergman (1966), is most likely due to the misuse of fluid parameters n and K obtained at high shear rate in analyzing low flow rate experimental data. Secondly, the measured profile was graphically integrated and compared with the measured average velocity (volumetric flow rate/cross-sectional area). Without exception, both methods give the accuracy of point velocity measurements well within the $\pm$ 5% confidence limit.

Entrance Velocity Profile Following an Abrupt Contraction

Three fluids (VS1, VS2, and VS3) were used to examine the entrance velocity profile following the abrupt 2:1 annular contraction. Figure 1 shows the entrance velocity profile for fluid VS1-3 at a distance of 0.9 cm downstream of the contraction. This fluid has the lowest elasticity number, ξ = 0.0076, among all the test fluids at different experimental conditions. Despite the low elasticity and high Reynolds number, Re = 973, the velocity profile is already partially developed (89% of fully developed $u_{mf}/<u>$) very near the contraction plane. At the same Reynolds number and position, the profile for inelastic fluids would have been quite flat. This indicates that the predevelopment of the flow field upstream of the contraction is due solely to fluid elasticity. The theoretical profile shown in Figure 1 was obtained from the boundary-layer solution, which assumed a flat entry profile (Tiu and Tan, 1977).

For fluids with higher elasticity, as characterized by large ξ, the entrance velocity profile was observed to be fully developed. Table 1a tabulates the percentage development of the entrance maximum velocity as a function of ξ in the abrupt entry geometry for all the experimental runs carried out with fluids VS1, VS2, and VS3. It is seen that for $\xi < 0.03$, the entrance velocity profile is partially developed. It is fully developed in the range $0.0307 \leqq \xi \leqq 0.296$, and $23 \leqq Re \leqq 210$. It is not possible to separate the effect of fluid elasticity from axial diffusion of momentum in the low Reynolds number region unless corresponding measurements are made for inelastic fluids at the same Reynolds number. At very low Reynolds numbers ($Re < 10$), it is interesting to note that overdevelopment of the velocity profile at the entrance was observed for VS3-1 (13%). This interesting phenomenon will be discussed later.

No lower limit of ξ, which marks the onset of velocity predevelopment, can be established from the experimental results. Theoretically, any observed fluid elasticity could result in some development of velocity at the entrance.

The concept of hydrodynamic entry length as being the axial distance required for the velocity profile to develop from an uniform (flat) shape at the entrance to the fully developed shape is, therefore, not strictly valid for viscoelastic fluids, since the flow field is already partially developed at the entrance. It is very important, then, to insure that the initial condition is the same for both viscoelastic and inelastic fluids before their measured entry lengths can be compared.

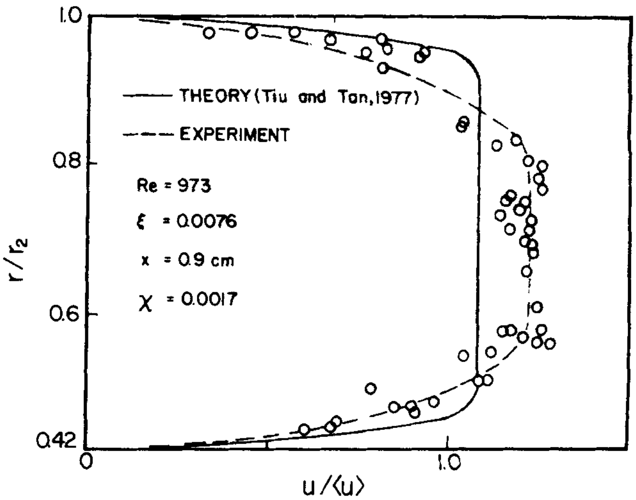

Figure 1. Velocity profile for fluid VS1-3 near the contraction of an abrupt entry.

Entrance Velocity Profile Following a Smoothly Converging Conical Section

Two viscoelastic fluids, VS4 and VS5, were used to investigate the effect on the entrance velocity profile of a conical section upstream of the contraction. Percentage development of the entrance velocity profile is given in Table 1b. It appears that the conical section has very little effect on the measured entrance velocity profile. The magnitudes of percentage development are similar to those for the abrupt entry. For example, fluid VS4-1 at ξ = 0.137 shows a 100% fully developed entrance velocity profile which is consistent with any fluids where $\xi > 0.03$ in an abrupt entry flow. Unfortunately, it was not possible to make a more quantitative comparison between the two sets of data, as it was difficult to measure the velocity profiles in both geometries at either the same ξ or the same axial distance.

Velocity Overdevelopment

At very low Reynolds numbers, the maximum velocity at the entrance is seen to overdevelop; it is larger than the fully developed value in both the abrupt and conical entry geometries (fluids VS3-1 and VS5-1 in Table 1). Figure 2a is a plot of the entrance velocity profile for VS3-1 in the abrupt entry geometry at Re = 2.53 and ξ = 1.761. The theoretical fully developed velocity profile (Fredrickson and Bird, 1958) is included for comparison. In contrast to the rather blunt central core of the fully developed profile, the measured velocity profile is more pointed. The measured $u_m/<u>$ is approximately 13% higher than $u_{mf}/<u>$.

An overdeveloped velocity profile was also obtained for fluid VS5-1 in the conical entry. The condition occurred at a lower value of elasticity number (ξ = 0.573) and a higher Reynolds number (Re = 8.2). The velocity profile is shown in Figure 2b. Although this fluid was not as elastic as VS3-1 in Figure 2a, the degree of overdevelopment was also found to be approximately 13%. The profile is not as skew as the one in an abrupt entry. This is expected, since the transition of velocity field through a conical entry is more gradual and smooth.

A recent experimental investigation by Nakamura et al. (1976) lends support to the observed overdevelopment of the viscoelastic flow field at the entrance. At Re = 4.3, the center-line velocity at the entrance to a 3.5 to 1 abrupt circular contraction was found to be 20% overdeveloped.

TABLE 1. PERCENTAGE DEVELOPMENT OF THE ENTRANCE VELOCITY

Fluid*	ξ	Re	$u_m/<u>$ as a percentage of $u_{mf}/<u>$	Distance downstream of entrance (cm)
a. Abrupt entry geometry				
VS1-1	0.0171	342	97	0.9
VS1-2	0.0134	522	92	0.9
VS1-3	0.0076	973	89	0.9
VS2-1	0.0307	210	100	0.7
VS2-2	0.0146	532	98	0.7
VS2-3	0.0109	764	95	0.8
VS3-1	1.761	2.53	113	0.3
VS3-2	0.296	23.3	100	0.2
VS3-3	0.051	149	100	0.3
b. Conical entry geometry				
VS4-1	0.137	28	100	0.2
VS4-2	0.0152	517	90	0.2
VS5-1	0.573	8.2	113	0.2
VS5-2	0.0245	420	92	0.2

* Note that fluids are coded for identification purposes in letters and numbers according to nominal concentrations and experimental runs.

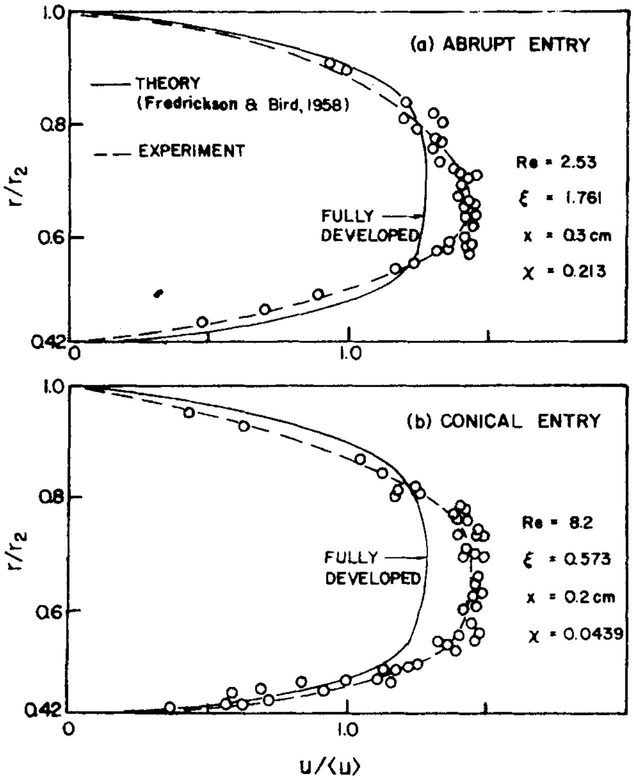

Figure 2. Overdevelopment of velocity profiles near the contraction. *a*. Abrupt entry. *b*. Conical entry.

Their larger percentage overshoot, compared to this work, is most likely due to the use of a larger contraction ratio (3.5 to 1 as compared to 2 to 1). A viscoelastic aqueous solution of 0.4 wt % sodium polyacrylate was used. Although no elastic property measurement was reported, the flow behavior index n was 0.46. The fluid is thus of quite similar shear thinning behavior as fluids VS3-1 (n = 0.343) and VS5-1 (n = 0.342).

At low Reynolds numbers, the flow field for an inelastic fluid has been predicted and observed to start developing before the contraction owing to the axial diffusion of momentum (Vrentas and Duda, 1967). However, no overdevelopment of the flow field at the entrance has been reported. The results of this investigation and of Nakamura et al. (1976) indicate that in the creeping flow limit, high fluid elasticity has an additional effect, over and above that due to axial diffusion of momentum, on the predevelopment of the flow field in the upstream region.

The experimentally observed overshoot in the core velocity at the entrance region for pipe and annular geometries has also been predicted for an Oldroyd type of fluid in a plane two-dimensional flow (L and T shaped and slit) with abrupt changes in geometry (Perera and Walters, 1977). Unfortunately, the parameters used in the analysis were not of the right form to allow the equivalent Reynolds and Weissenberg numbers to be calculated. In addition, the axial velocity near the wall was predicted to develop in an oscillatory manner. This could not be verified here for the annular geometry owing to the experimental limitations.

A better understanding of the mechanism involved in the overshoot can be obtained from a study of the flow pattern in the upstream region. Streakline photographs of the upstream flow pattern were also taken for fluids at similar flow conditions when inlet velocity profiles were measured. A large stationary vortex was observed in the 90 deg corner just upstream of the contraction plane for the abrupt entry geometry. The bulk flow converges smoothly into the downstream annulus through a funnel shaped region formed by the stationary vortex. There is no transfer of fluid elements between the vortex and the bulk flow region. To satisfy the equation of continuity, the bulk of the fluid has to accelerate through the funnel shaped region into the smaller annulus. The acceleration is maximum at the contraction plane. Very near the solid and stationary vortex boundaries, the fluid motion is primarily due to shearing flow. At the core of the funnel shaped region near the inner wall of the annulus, the flow field is similar to elongation flow, with fluid elements being pulled into the smaller annulus, causing an overshoot of core velocity over its fully developed value. At low Reynolds numbers, the fluid relaxation time is sufficiently large such that any tensile stresses created in the funnel shaped region are not able to relax completely when the fluid reaches the contraction plane. In the context of viscoelastic fluids with fading memory, they are said to remember their past stress history. Hence, the velocity profile in the funnel shaped region upstream would still be sustained at the contraction plane.

A similar type of flow pattern is observed with the conical entry geometry. As far as the fluid in the core of the bulk flow region is concerned, the conical section is no different from a stationary vortex. This could be the reason why results for the conical entry geometry are in agreement with those for the abrupt entry, since flow patterns upstream of the contraction plane have essentially the same features for both geometries.

ACKNOWLEDGMENT

The financial support received from the Australian Research Grants Committee is gratefully acknowledged.

NOTATION

K, n, σ, s = power law parameters

N_1 $= P_{11} - P_{22}$ = first normal stress difference in a viscometric flow

r = radial position
r_1, r_2 = inner and outer radii of the annulus
r_H = $(r_2 - r_1)/2$ = hydraulic radius
Re = Reynolds number
u = local velocity in the x direction
u_m = maximum velocity in the entrance region
u_{mf} = maximum velocity in the fully developed flow
$\langle u \rangle$ = average velocity
Ws = $(\sigma/K)(\langle u \rangle / r_H)^{s-n}$, Weissenberg number
x = axial distance measured from the contraction plane
χ = $x/r_H Re$, dimensionless axial distance
ξ = Ws/Re = elasticity number = elastic force/inertial force
$\dot{\gamma}$ = $-du/dr$, shear rate in steady shear flow
τ = shear stress

LITERATURE CITED

Ballenger, T. F., and J. L. White, "An Experimental Study of Flow Patterns in Polymer Fluids in the Reservoir of a Capillary Rheometer," *Chem. Eng. Sci.*, **25**, 1191 (1970).
Boger, D. V., and A. V. Ramamurthy, "Flow of Viscoelastic Fluids Through an Abrupt Contraction," *Rheol. Acta.*, **11**, 61 (1972).
Brocklebank, M. P., and J. M. Smith, "Developing Laminar Flows with Viscoelastic and Non-Newtonian Liquids," *ibid.*, **9**, 396 (1970).
Busby, E. T., and W. C. MacSporran, "An Experimental Study of Non-Rheometric Flows of Viscoelastic Fluids: I. Flow Into a Re-Entrant Tube," *J. Non-Newtonian Fluid Mech.*, **1**, 71 (1976).
Cable, P. J., and D. V. Boger, "A Comprehensive Experimental Investigation of Tubular Entry Flow of Viscoelastic Fluids," *AIChE J.*, **24**, 869 992 (1978), and **25**, 152 (1979).
Fredrickson, A. G., and R. B. Bird, "Non-Newtonian Flow in Annuli," *Ind. Eng. Chem.*, **50**, 347 (1958); Erratum, *Ind. Eng. Chem. Fundamentals*, **3**, 383 (1964).
Nakamura, R., N. Yoshioka, M. Higuchi, and H. Hira, "Velocity Profile in Viscoelastic Flow at the Entry of a Tube," *J. Chem. Eng. (Japan)*, **9**, 291 (1976).
Perera, M. G. N., and K. Walters, "Long-Range Memory Effect in Flows Involving Abrupt Changes in Geometry: Parts I and II," *J. Non-Newtonian Fluid Mech.*, **2**, 49, 191 (1977).
Sutterby, J. L., "Laminar Converging Flow of Dilute Polymer Solutions in Conical Sections II," *Trans. Soc. Rheol.*, **9**, 227 (1965).
Tan, K. L., and C. Tiu, "Entry Flow Behaviour of Viscoelastic Fluids in an Annulus," *J. Non-Newtonian Fluids Mech.*, **3**, 25 (1977).
Tiu, C., and K. L. Tan, "Boundary Layer Analysis of Viscoelastic Flow in Annuli," *Rheol. Acta*, **16**, 497 (1977).
Tiu, C., and S. Bhattacharyya, "Developing and Fully-Developed Velocity Profiles for Inelastic Power-Law Fluids in an Annulus," *AIChE J.*, **20**, 1140 (1974).
Tordella, J. P., "Capillary Flow of Molten Polyethylene—A Photographic Study of Melt Fracture," *Trans. Soc. Rheol.*, **1**, 203 (1957).
Vaughn, R. D., and P. D. Bergman, "Laminar Flow of Non-Newtonian Fluids in Concentric Annuli," *Ind. Eng. Chem. Process Design Develop.*, **5**, 44 (1966).
Vrentas, J. S., and J. L. Duda, "Effect of Axial Diffusion of Vorticity on Flow Development in Circular Conduits: Part 2. Analytical Solutions at Low Reynolds Number," *AIChE J.*, **13**, 97 (1967).

Manuscript received August 7, 1978; revision received November 22, 1978 and accepted January 11, 1979.

Periodic Flow in a Curved Tube

J. Y. LIN and J. M. TARBELL

**Department of Chemical Engineering
Pennsylvania State University
University Park, Pennsylvania 16802**

Steady Poiseuille flow in a rigid curved tube and its effect on associated transport processes have been studied extensively since Dean (1927) first predicted the characteristic twin vortex secondary motion (Figure 1a) associated with this flow (see, for example, Tarbell and Samuels 1973, Kalb and Seader 1974, and Smith 1976). However, periodic Poiseuille flow in a rigid curved tube was first analyzed only recently by Lyne (1970) who considered the problem of pure sinusoidal flow, with zero mean. Lyne constructed an asymptotic expansion solution of the Navier-Stokes equations, valid for high frequencies, $\alpha >> 1$, and obtained a striking result. For sufficiently high frequencies ($\alpha > 12.9$), the twin vortex motion which characterizes steady flow transforms to a qualitatively new four-vortex motion (Figure 1b). Inward centrifuging near the center of the tube was quite unexpected. Independent theoretical studies of this same problem by Zalosh and Nelson (1973) and Chandran (1974), employing quite distinct solution techniques, predicted the same new four-vortex motion. These theoretical predictions of the new phenomenon were subsequently verified experimentally by Bertelsen (1975).

In this note, we report numerical results on the problem of periodic Poiseuille flow (*with non-zero mean*) in a rigid curved tube. The only previous work on this problem was by Smith (1975), who used a variety of asymptotic expansions to obtain approximate solutions in the high ($\alpha >> 1$) and low ($\alpha << 1$) frequency regimes. We believe that our numerical results for intermediate frequencies reveal a significant new phenomenon—*resonance between the axial flow and the secondary flow.* This means that the secondary flow has a natural frequency. approximately equal to the circulation time of the secondary flow at the time-averaged flow rate, which is excited by the oscillating axial flow over a narrow range of frequencies. The character of the resonating flow may be quite unusual and not anticipated on the basis of low frequency (quasi-steady) or high frequency (relaxed steady) information. It must be emphasized that the four-vortex secondary flow discovered by